



Tips for Teachers

Invite students to make reasoned decisions about virtually every aspect of mathematics.

Critical thinking in elementary mathematics: What? Why? When? and How?

What is critical thinking in mathematics?

When students think critically in mathematics, they make reasoned decisions or judgments about what to do and think. In other words, students consider the criteria or grounds for a thoughtful decision and do not simply guess or apply a rule without assessing its relevance. For example, rather than directing students to use a particular strategy to solve an assigned problem, the teacher works with them to identify various strategies and to develop criteria for choosing a suitable strategy from among the options.

In the classroom

A grade four class worked with their teacher to develop these criteria for an effective problem-solving strategy:

- helps me understand the problem
- doesn't take too long
- leads to a reasonable answer
- makes me feel confident

Presented with a challenging problem, students used the criteria to help them choose a strategy. Would they model with objects, draw a diagram, make a list or create a table? In a follow-up discussion, students used the criteria to justify or critique their choice of strategies. The teacher emphasized that we often need to "play around" with a strategy before deciding whether or not it is effective and that no single strategy is best in every situation or for every person. She posted the criteria in a prominent place for on-going reference.



Why take the time to promote critical thinking in mathematics?

Time invested in developing critical thinking pays off when students “learn to think and think to learn.” Students who are critically thoughtful in mathematics develop:

- **deeper engagement and understanding.** Research and common sense tell us that, no matter how hard we try, we can not think or understand for our students. We can, however, create conditions that encourage students to “turn on” their brains and actively engage in learning mathematics through critical inquiry.
- **greater independence and self-regulation.** By helping students develop a repertoire of thinking tools that they are able to use independently, we can support their growing confidence in thinking for themselves and monitoring their own learning.
- **stronger competence with mathematical processes.** Current standards in math education call for a focus on problem solving, reasoning, representing and communicating. Each of these processes (or “mathematical practices”) is strengthened when students think critically about mathematics.

In the classroom

A kindergarten class gathered on the carpet for their daily “number talk”. The children focused intently as their teacher held up a rectangular “dot card” with five dots on the left and two dots on the right. From previous sessions, the students knew that their task was not only to tell the number of dots on the card, but also to explain how they had reached their answer. The teacher waited until most students were giving a silent “thumbs-up” signal before she called on one boy to come to the front to share his thinking.

“I know that there are seven because I counted them all,” he said, pointing to one dot at a time. “1, 2, 3, 4, 5, 6, 7.”

“How many people used this counting strategy?” the teacher asked and several hands shot up. The teacher probed further. “Is this a useful strategy for dot cards? How do we know?”

“Yes – because it gives the right answer,” said one girl. Others nodded in agreement, recognizing the single criterion they had previously developed with their teacher. One at a time, the teacher invited two more students to share their strategies. One girl explained that she recognized the five dots on one side “because it’s a pattern like on dice” and then counted up, “5, 6, 7.” A boy showed the class how he counted the vertical pairs of dots by twos, “2, 4, 6,” and then added the single dot from the center of the 5 pattern. Throughout the number talk, the teacher gently encouraged students to assess the usefulness of each strategy. After class, she reflected that her students now seemed ready to use two criteria and made plans to introduce “uses a pattern” the next day.



When should we invite critical thinking?

If a critically thoughtful approach helps students better understand what they are learning, it makes sense to invite students to make reasoned decisions about virtually every aspect of mathematics, including:

- selecting strategies for building number sense and mastery of basic facts
- deciding how to approach a problem for which they have no ready-made solution or procedure
- choosing the most appropriate way to represent a mathematical situation
- monitoring their problem solving progress and adjusting as necessary
- analyzing their own responses and asking, "Does this make sense?"
- communicating their mathematical ideas effectively
- connecting mathematics with their own lives and the wider world

In the classroom

"Imagine you are the manager of a new NHL team," a teacher told her hockey-crazed grade five class. "Use the statistics to choose one of these three players and then create a convincing graph to persuade the team owners you have made the best choice." After an animated discussion, the class decided that a valuable player would be someone who scored many goals, made lots of assists, played in nearly every game and didn't spend much time in the penalty box. As they scoured the stats, students realized that they would need to weigh the evidence carefully as they made their choices.

Moving into the second part of the challenge, the class agreed that a convincing graph would be focused, well organized, clearly labeled and appropriate for the type of data being displayed. Over two days, the students made their decisions and prepared their graphs. Excitement ran high as they shared their work and defended their choices. Later, the teacher commented, "For some, it was probably the first time they engaged in thinking deeply about math."

How does the TC² framework support critical inquiry in mathematics?

The TC² framework suggests that teachers help students improve as critical thinkers by:

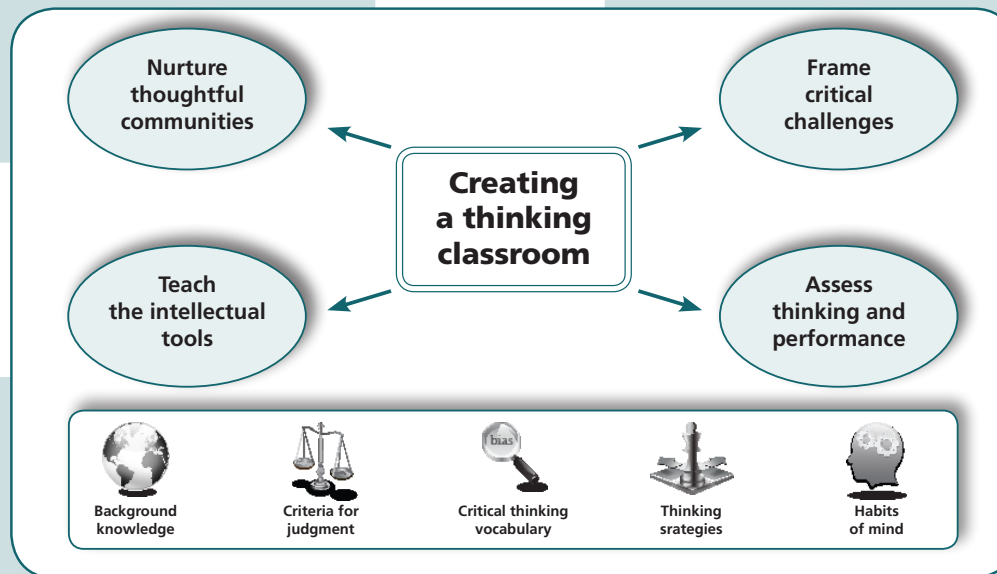
- nurturing communities of thinkers
- framing critical challenges that invite students to think critically
- teaching the intellectual tools that enable students to address critical challenges
- assessing thinking and performance to support students' use of the intellectual tools



The following chart suggests a few ideas that teachers might try in elementary math for each front of the TC² framework.

- deliberately nurture a classroom culture in which mathematical discussion is part of the daily routine
- start with brief “number talks” during which students share different mental math strategies for carrying out a given computation
- draw the class together after a problem solving session to share, discuss and analyze the various strategies they used
- set clear expectations for respectful participation in discussions (e.g., have students ask for clarification when they disagree with another student’s response)

- use different types of critical challenges to frame engaging and thought-provoking math tasks (e.g., Critique the piece, Judge the better piece, Decode the puzzle)
- present problems for which students have no predetermined solution strategies
- present “open” problems that have more than one reasonable solution
- challenge students to find and use mathematics in real world situations
- encourage investigation of multiple solution strategies



- co-construct criteria with students (note that some criteria will be used again and again)
- teach flexible thinking strategies such as analyzing a word problem by identifying “what I know” and “what I need to find out”
- foster helpful habits of mind such as perseverance through stories, explicit encouragement, role play and modeling
- develop background knowledge through critical inquiry (e.g., give examples and non-examples of quadrilaterals and ask students to figure out the sorting rule)

- use co-constructed criteria to support self-assessment and peer feedback
- offer various ways for students to provide evidence of their thinking (e.g., as well as using pictures, numbers and words on paper, students might demonstrate their ideas in an oral interview, or by creating a skit, poem, poster, blog post or concrete model)
- ensure that your assessment reinforces the value of “showing your work” and “explaining your thinking”